

EQUILIBRATION AND OUT-OF-EQUILIBRIUM EFFECT IN RELATIVISTIC HEAVY ION COLLISIONS

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The approach of a parton plasma at future heavy ion colliders towards kinetic and chemical equilibrium is considered. A plasma with a self-consistent evolving parton-parton interaction strength is shown to equilibrate better and faster than the usual but inconsistent one with a fixed strength. We explain why as a consequence of this, a parton plasma is a unique kind of many-body system. Because our time evolution scheme does not require the plasma to be in either kind of equilibrium from the outset, out-of-equilibrium effect on particle productions can be revealed. We show this on photon production and discuss the implications on photon as a signal to detect the quark-gluon plasma. **IASA 98-3, UA/NPPS-98-13**

1 Introduction

The trophy of the game of relativistic heavy ion collisions at the future Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) is well known, namely the quark-gluon plasma. To reach this goal, one interrogates the particles coming out from the beginning till the very end of the collisions. Based on the information thus obtained, one determines whether a phase transition into the predicted nearly free quarks and gluons state has occurred. In order to perform the necessary analysis, qualitative understanding and quantitative control are very important. In this talk, we present some recent development in the former. We will look at the equilibration of the parton plasma in the following sections to determine the state of the system just before the phase transition. One of the reasons being that if the system is able to complete the equilibration process, then it would be much simpler to describe. The formalism of thermal field theory for QCD would then be applicable. If the system is, on the other hand, still out of equilibrium, this much more complicated situation would require a phenomenological approach

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as the out-of-equilibrium thermal field theory is not yet fully developed. Another reason is the state of equilibrium of the system has important effects on the particle emissions or their production rates. We will show this effect on photon production from the plasma. Before we do that, the parton plasma as a rather unique many-body system will be shown first.

2 Equilibration of a parton plasma — fixed vs. evolving α_s

To study the equilibration of the parton plasma, we must have a means of performing the time evolution of such a many-body system. To do this, we choose some plausible initial conditions from event generator such as HIJING. Then we build the time evolution equations from Boltzmann equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_p \cdot \frac{\partial}{\partial \mathbf{r}} \right) f(p, \tau) = C(p, \tau) . \quad (1)$$

The collision terms $C(p, \tau)$ will be constructed in two ways in order to close the equations. The first is to use the relaxation time approximation and the second is by explicit construction from QCD matrix elements and the particle distributions f_g , f_q and $f_{\bar{q}}$. We include all binary interactions at the leading order in the renormalized strong coupling

$$\begin{array}{lll} gg \longleftrightarrow gg & qq \longleftrightarrow qq & \bar{q}g \longleftrightarrow \bar{q}g \\ qq \longleftrightarrow qq & q\bar{q} \longleftrightarrow q\bar{q} & \bar{q}\bar{q} \longleftrightarrow \bar{q}\bar{q} \end{array} \quad (2)$$

and in order to have non-conservation of the number of partons, which one would expect from interactions generated by the non-Abelian QCD Lagrangian, we include also

$$gg \longleftrightarrow ggg \quad gg \longleftrightarrow q\bar{q} . \quad (3)$$

Combining all these ingredients, we can solve for the distributions, which are functions of time and from which most information of the plasma can be obtained and therefore the time evolution is known. Of course, it is still necessary to choose a most appropriate value of the coupling to evaluate the collision terms. For this purpose, it is customary to assume an average momentum transfer of 2 GeV for the parton interactions in the plasma. This translates into an $\alpha_s = 0.3$ for $\Lambda_{QCD} = 200$ MeV. With this choice, the system can be evolved in time and one can check for the state of equilibrium. Concerning the latter, there are two aspects. One is the balance of the partonic composition in the plasma or parton chemical equilibration. The degree of chemical equilibrium is most simply parametrized by the quantity known as the fugacity which we denote here by l_g and l_q for gluons and quarks respectively. The plasma

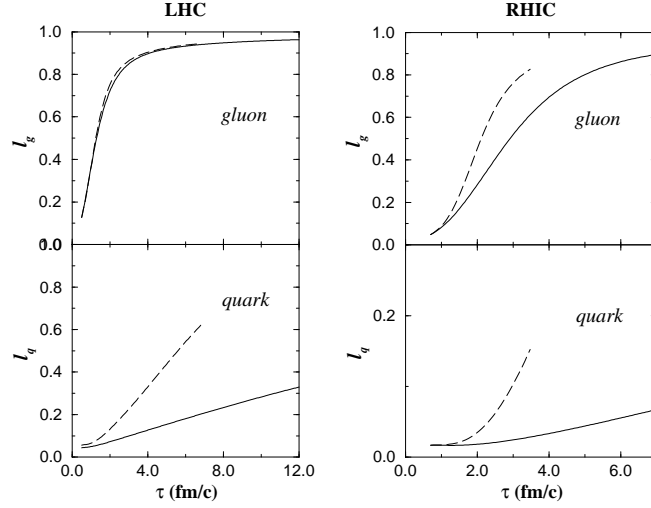


Figure 1: Variation of the gluon and quark fugacities, l_g and l_q , with time. The solid (long dashed) lines are the results of fixed (evolving) α_s .

is in chemical equilibrium if $l = 1.0$ or very far from equilibrium if $l = 0.0$. In Fig. 1, the results of the change of the fugacities with time at LHC and at RHIC are shown in solid lines. As can be seen, chemical equilibration for gluon is very good at LHC and good at RHIC. On the contrary, that for quarks at both colliders is not so good and even poor at RHIC.

As for thermalization or kinetic equilibration, there is no one parameter as in chemical equilibration to parametrize the degree of thermalization, so an indicator of the degree of thermalization must be found. Usually, this is done by examining the slope of the particle p_T -spectra or that of the log of the particle distributions. In our case, we choose instead to plot the longitudinal to transverse pressure ratios because the pressure must be the same in all directions when the system is in kinetic equilibrium. So the closer are these ratios to unity, the nearer is the parton system to full thermal equilibrium. In Fig. 2, we plot these ratios in solid lines for gluon and quark separately. The particular shape of the curves has to do with our rather special initial conditions, which we take to be momentarily thermalized at the beginning. Therefore these curves all start with a ratio of 1.0. As the expansion sets in, the system is driven out from the initial transient kinetically equilibrated state and the pressures are no longer isotropic. The system naturally responds to the expansion and increases the collision rate. At some point, the latter dominates

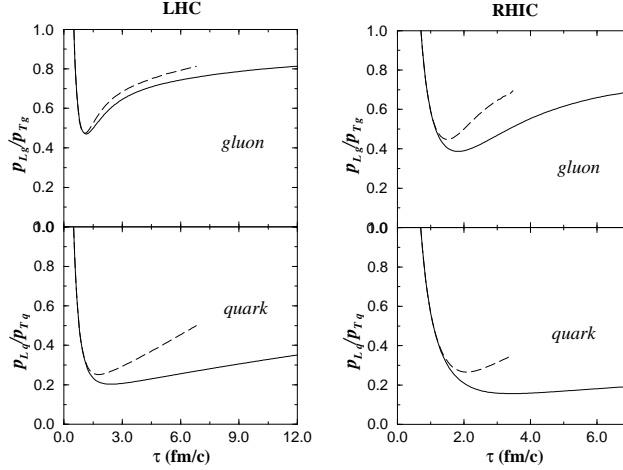


Figure 2: Variation of the ratios of longitudinal to transverse pressure for gluon and quark with time. The solid (long dashed) lines are the results of fixed (evolving) α_s .

over the expansion and the curves all approach unity again after the dip. We are more interested in the final approach to kinetic equilibrium. As can be seen, the gluons are better equilibrated than the quarks, which are again not so good.

So after we used some reasonable initial conditions, an essentially perturbative time evolution scheme and an appropriate value of the coupling, the state of equilibrium of the parton phase is reasonably good for gluons but not so good for the quarks. One wonders whether it is possible to do better than this. Evidently, the closer is the plasma to full equilibrium the better. In fact, the answer is yes. Improvement is possible because what we have done so far, although apparently reasonable, is not entirely correct for a time-evolving system. It is this last aspect, which is unusual within the framework of perturbative calculations of strong interactions, that one could easily overlook^{1,2,3,4} and indeed has been the case until recently⁵.

In fact, during the time evolution of the system that produced the previous results, the average parton energies dropped significantly by at least 1.0 GeV and so the system underwent substantial changes. It is therefore very unlikely that the average momentum transfer in an average parton collision can stay fixed at around 2.0 GeV throughout. If so, a way must be derived to replace the fixed value of $\alpha_s = 0.3$ used to obtain the previous results. A simple recipe naturally suggests itself, which is to take the average momentum transfer $\langle Q \rangle$

to be given by the average parton energy $\langle E \rangle$. Then one substitutes this into the one-loop running coupling expression

$$\alpha_s(\langle E \rangle) = \frac{4\pi}{\beta_0 \ln(\langle E \rangle^2 / \Lambda_{QCD}^2)} \quad (4)$$

to obtain an interaction strength entirely determined by the system. As a consequence, this strong coupling will evolve with the plasma and as such this new approach is more self-consistent.

The new results of the time evolution scheme with the self-consistent coupling are the long dash curves in Fig. 1 and Fig. 2. The dashed curves in Fig. 1 and those in Fig. 2 during the final approach to equilibrium all rise faster with time than when the plasma is evolved with a fixed α_s (solid line). Not only is that the case, the final state of equilibrium is markedly better than before at both colliders. So a self-consistent time evolution speeds up and improves the equilibration of the parton plasma. There are other associated effects on the plasma that go along with the aboves but we shall not discuss them here. We refer the reader to⁵.

3 SU(3) non-Abelian parton plasma is an unique many-body system

In the previous section, we saw how the evolving coupling improved the equilibration of the parton plasma. This is but a manifestation of an unique property of a many-body system governed by the QCD Lagrangian. In the Boltzmann equation, it is the collision terms that are responsible for bringing the plasma into equilibrium. The collision terms are all made up of the difference between the reactions going one way and in the reverse direction. In an ordinary many-body system, as equilibrium is approached, the forward and backward reaction rates get closer and closer to each other and the equilibration rate will become slower and slower as a result. On the contrary in a parton plasma, because the interactions are mediated via SU(3) non-Abelian gauge bosons, the interaction strength varies with the scale of the interactions. The collision terms of such a system are therefore each given by a certain power of a varying α_s multiplied by the difference of the rate of the forward and backward reaction. As equilibrium is approached, the reaction rates in both directions tend to equalize, however, the equilibration rate of a parton plasma does not slow down, unlike an ordinary plasma, because the increasing strength of the interactions caused by the lowering of the average energy of the system is able to more than compensate for the otherwise equalization of the forward and backward reactions or in other words, the slowing down of the equilibration rate. The manifestation of this effect can be most easily seen on the variation of the collision

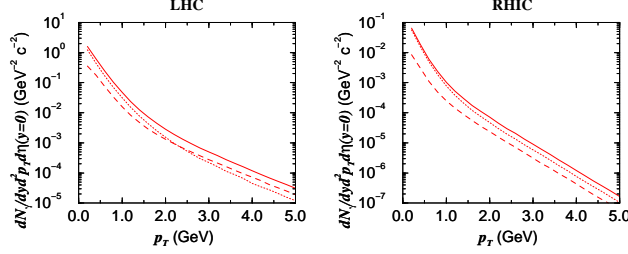


Figure 3: Photon production at LHC and at RHIC from a parton plasma not in equilibrium. The dotted (dashed) lines are from Compton scattering (annihilation) contribution. The solid lines on the top are the total contributions. The contribution from Compton scattering does not dominate over that from annihilation at LHC.

time with time (see ⁵). Therefore the parton plasma exhibits a most unusual feature of the tendency to equilibrate faster and faster. Obviously, this cannot continue forever and at some point the equilibration of the quarks and gluons will be interrupted by the start of the deconfinement phase transition.

4 Out-of-equilibrium effect on photon production

We have shown the time evolution of a parton plasma at the two colliders. All the information of the system is contained in the particle distributions and from which we learned about the plasma's state of equilibrium. With the knowledge of the momentum distributions of the particles, one can work out the rates of particle production such as photons, dileptons etc.. Also our method of evolving the plasma in time, imposed no equilibrium requirement on any aspects of equilibration and hence the form of the distributions. As such, a direct comparison of particle productions of a plasma that is out of equilibrium with an equilibrated one can be done. We will show an out-of-equilibrium effect on photon production and explain its importance.

Photon production from the plasma is dominantly through Compton scattering and annihilation contribution. The total production rate from these two contributions is given by

$$\begin{aligned}
 E \frac{d^7 N}{d^3 p d^4 x} &= \frac{1}{2(2\pi)^3} \int \frac{d^3 \mathbf{k}_1}{(2\pi)^3 2\omega_1} \frac{d^3 \mathbf{k}_2}{(2\pi)^3 2\omega_2} \frac{d^3 \mathbf{k}_3}{(2\pi)^3 2\omega_3} (2\pi)^4 \delta^{(4)}(k_1 + k_2 - k_3 - p) \\
 &\times \left\{ 2f_g(k_1, \tau) f_q(k_2, \tau) (1 - f_q(k_3, \tau)) |\mathcal{M}_{gq \rightarrow q\gamma}|^2 \right. \\
 &\quad \left. + f_q(k_1, \tau) f_{\bar{q}}(k_2, \tau) (1 + f_g(k_3, \tau)) |\mathcal{M}_{q\bar{q} \rightarrow g\gamma}|^2 \right\}. \quad (5)
 \end{aligned}$$

In ref. ⁶, a kinetically equilibrated plasma was studied and the photon rate calculated. It was shown in their Fig. 8 and 9 that the contribution from Compton scattering was far more important than the annihilation contribution both at LHC and at RHIC energies. If one now compares these figures with the one we have here in Fig. 3, one can immediately see that in our plots, Compton scattering contribution does not dominate over that of annihilation. In fact at higher p_T at LHC, annihilation contribution is the more important. At RHIC in the same p_T range, although Compton scattering is larger, it is not by the same large amount as shown in ref. ⁶. The reason for these differences between our photon production and those in ref. ⁶ is because our plasma is not in kinetic equilibrium but that in ref. ⁶ was. We will explain this in the next paragraph.

In Eq. (5), the Compton and annihilation contribution to photon yield differ by various factors such as the interaction matrix elements, the Pauli blocking and Bose-Einstein stimulated emission, and one of the initial distributions. In fact, the matrix elements do not differ significantly numerically and the Pauli blocking and Bose-Einstein stimulated emission also provide no great suppression or enhancement. The source responsible for the difference seen in the two different sets of figures comes actually from the gluon distribution in Compton scattering and the initial quark distribution in the annihilation contribution. To explain this more clearly, we use the observation that the difference in the figures is most prominent for larger p_T photons — annihilation contribution is actually larger than Compton scattering at LHC in Fig. 3 — so we can concentrate in this p_T range. Emission of larger p_T photons requires more energetic incoming partons, this let us simplify the explanation by assuming Boltzmann form for the incoming particle distributions $f \sim l \exp\{-k^0/T\}$, while for the outgoing ones, we will replace them by unity. In reality, Pauli blocking and Bose-Einstein stimulated emission do have some effects but they are not so important in comparison with the one that we are going to describe so it is best to leave them out. Since we are comparing the two contributions, we form the ratio

$$\frac{\text{Annihilation}}{\text{Compton}} \sim \frac{l_q}{l_g} \exp\{k^0(1/T_g - 1/T_q)\} \quad (6)$$

after taking $|\mathcal{M}_{gq \rightarrow q\gamma}| \sim |\mathcal{M}_{q\bar{q} \rightarrow g\gamma}|$. Because of the initial gluon dominance and the stronger interaction amongst gluons due to colour, $l_q/l_g < 1$ always and therefore if the exponential factor is not present, Compton scattering indeed dominates over annihilation contribution. In a kinetically equilibrated parton plasma, the system has only one temperature $T_g = T_q$, so the exponential factor is unity and we get the results shown in Fig. 8 and 9 of ref.

⁶. In our present case, the plasma is not in kinetic equilibrium but we can most simply consider them as a mixed fluid of gluons and quark-antiquarks at different temperatures. The gluon temperature is higher initially but will eventually be lower than that of the quarks because of gluon multiplication and their conversion into quark-antiquark pairs. When that happens, for a photon with a high enough p_T , the incoming parton energies will be sufficiently large that the product in Eq. (6) will be larger than unity and the photon rate from annihilation will be larger than that from Compton scattering. This is what one sees in our result in Fig. 3 at LHC. In the same figure at RHIC, there is not sufficient time for the two temperatures to be in the correct range from each other so that the exponential factor is able to compensate for the fugacity ratio. The result is Compton scattering remains larger than the annihilation contribution but it is definitely by a lesser amount than that shown in Fig. 8 and 9 of ref. ⁶.

We see that the out-of-equilibrium effect tends to enhance the annihilation contribution at higher p_T and hence the total photon yield in that p_T range. Therefore photon production of partonic origin should have a better chance to compete with the hard photons from the initial collisions and those fragmented off minijets. The window for observing photons from deconfined matter will be widened as a result.

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